Stability and Bifurcation Analysis of Three-Species Predator-Prey Model with Non-monotonic Delayed Predator Response

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In this paper, we consider delayed three-species predator-prey model with non-monotonic functional response where two predator populations feed on a single prey population. Response function in both predator populations includes a time delay which represents the gestation period of the predator populations. Previous works of Collera [1, 2] also considered delayed three-species predator-prey model but with different response functions. In this study, we use delayed predator response as suggested in the paper of Ruan and Xiao [7]. We call a positive equilibrium solution of the form $E_*^s = (x*,y*,y*)$ as a symmetric equilibrium. The inspiration for this study are animal species that are similar but only differs in their gestation periods, e.g. Asian and Africa elephants. Another possibility is mutation within a single species where a part of the population is mutating causing them to have longer gestation periods. In [5], they show that in humans, children born at 37 weeks gestation have 1.5 IQ points lower that children born at 41 weeks. In [4], longer gestation period in elephants translates to better brain and cognitive capacity. Our results include conditions on the existence of equilibrium solutions, and stability and bifurcations of symmetric equilibria as the gestation periods of predator populations are varied. A numerical bifurcation analysis tool is also used to illustrate our results. Stability switch occurs at a Hopf bifurcation. Moreover, a branch of stable periodic solutions, obtained using numerical continuation, emerges from the Hopf bifurcation. This shows that the predator population with longer gestation period oscillates higher than the predator population with shorter gestation period.

Keywords: Predator-prey, non-monotonic, gestation, equilibrium, Hopf bifurcation

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