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Spontaneous Symmetry-Breaking in Deterministic Queueing Models with Delayed Information

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In queueing theory, a fluid model is a mathematical model which describes the fluid level in a reservoir where the periods of filling and emptying are randomly determined. Recently, a deterministic fluid model was constructed and analyzed in [4, 5] to describe the dynamical behavior of a system involving two queues. Their model incorporates customer choice behavior based on delayed queue length information. That is, the information given to the customers is based on the queue length information from a previous time and not on the current time. Waiting times in emergency rooms of hospitals, telephone call centers, and various rides in theme parks are some examples where in delayed information are provided to the customers. This time delay has an impact on the dynamical behavior of the queues and therefore has the capacity to affect the decision of a customer to choose which queue to wait in.

We generalize the queueing model studied in [4, 5] into N number of queues. The system of delay differential equations for this generalized model is equivariant under the finite symmetric group S_N. Spontaneous symmetry-breaking occurs in an equivariant dynamical system when the symmetry group of a solution of the equations is lesser than the symmetry group of the equations themselves [3]. In this work, we use group-theoretic techniques [1, 2] to show that the generalized model exhibits spontaneous symmetry-breaking. This, in turn, allows us to determine which type of solutions that can only occur in the system. In particular, we show that varying the time delay parameter can make a stable fully-symmetric equilibrium becomes unstable. This switch in stability occurs only at a symmetry-breaking Hopf bifurcation, that is, the branch of periodic solutions that bifurcates has lesser symmetry than the branch of fully-symmetric equilibria. Understanding such dynamical behavior of queues caused by providing delayed information to the customers provides insights that are beneficial for managers and operators of queueing systems.

References

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